



# MAPPING CONDITIONAL DISTRIBUTIONS FOR DOMAIN ADAPTATION UNDER GENERALIZED TARGET SHIFT ICLR 2022

Monday 25<sup>th</sup> April, 2022 to Friday 29<sup>th</sup> April, 2022

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# Introduction

## Unsupervised Domain Adaptation (UDA)

Train a good classifier  $h : \mathcal{X} \rightarrow \mathcal{Y}$  for an **unlabelled** target domain  $T$  given a labelled source domain  $S$ .

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# Problem definition

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## Generalized Target Shift

We consider the most challenging UDA setting, Generalized target shift (GeTarS) Zhang et al. 2013 where:

$$p_S(X|Y) \neq p_T(X|Y), p_S(Y) \neq p_T(Y)$$

# UDA under generalized target shift

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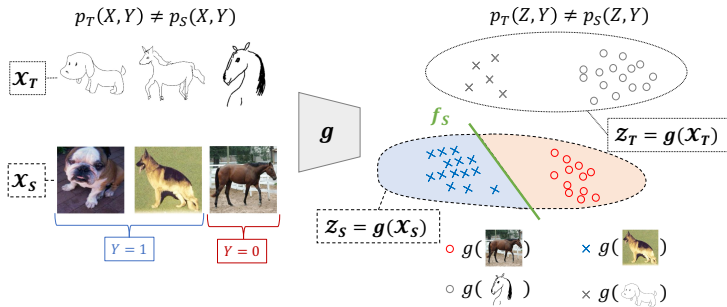
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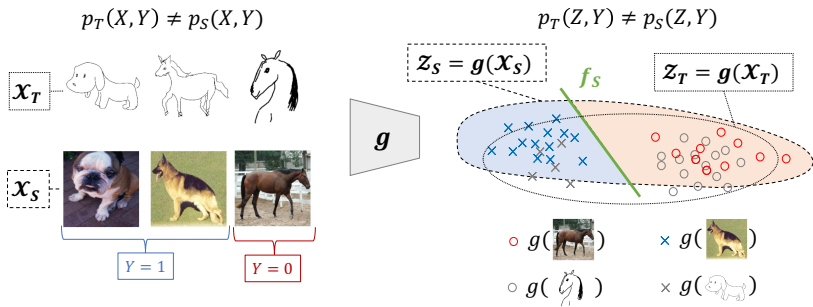


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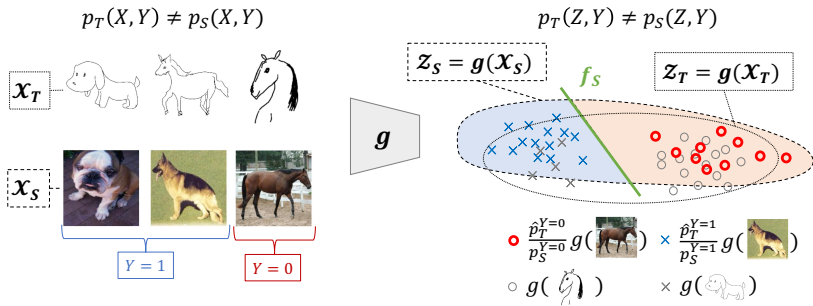
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- Learn  $f_S$  to classify encoded  $S$  samples.
- Learn  $g$  to match encoded  $S$  and  $T$  samples.
- Under GeTarS, reweight  $S$  samples by estimated class-ratios.

Combes et al. 2020; Gong et al. 2016; Rakotomamonjy et al. 2021; Shui et al. 2021



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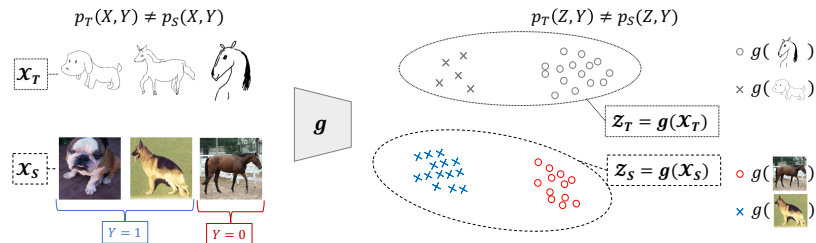
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    - unicity of the solution.

# OSTAR framework

# Optimal Sample Transport and Reweight (OSTAR) (I)

## Objectives

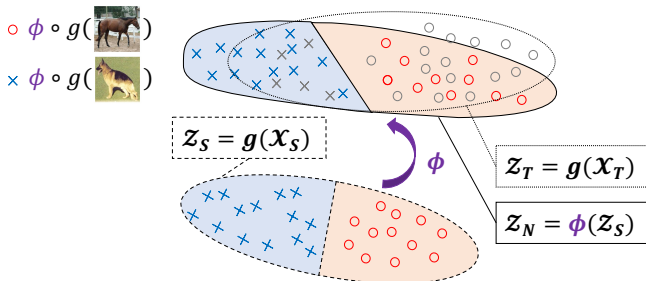
- Encode input  $S$  and  $T$  samples with  $g$



# Optimal Sample Transport and Reweight (OSTAR) (I)

## Objectives

- With fixed representations jointly
  - 1 Map encoded  $S$  samples onto  $T$  with  $\phi$  under OT constraints.

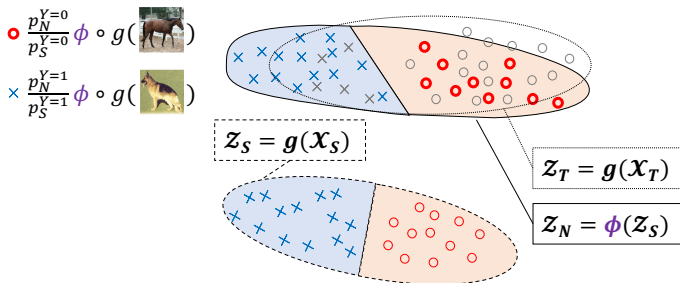


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2 Reweight mapped  $S$  samples with class-ratio estimates  $p_N^Y/p_S^Y$ .

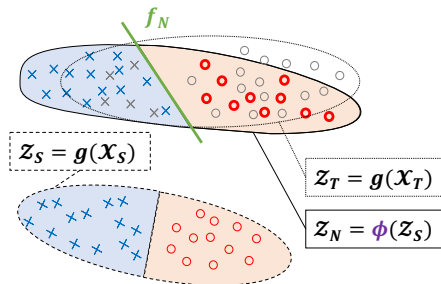




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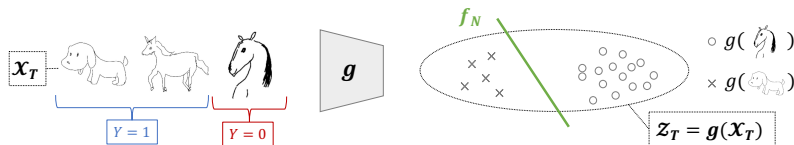
- With fixed representations jointly
- 3** Train classifier  $f_N$  on reweighted and mapped  $S$  samples



# Optimal Sample Transport and Reweight (OSTAR) (I)

## Objectives

- Use  $f_N$  for inference on encoded  $T$  samples.



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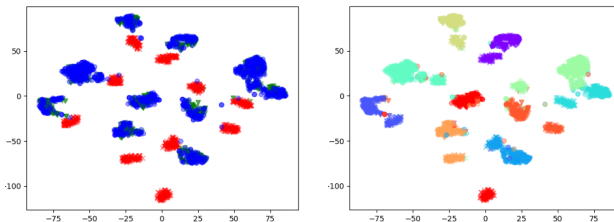
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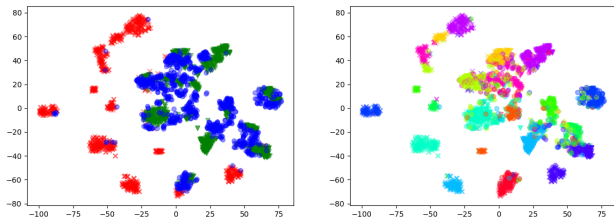
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- We define mild assumptions on  $g$ , for which
  - OSTAR controls the target risk at optimum.
  - Its solution  $(\phi, \mathbf{p}_N^Y)$  is unique.

# Results

# t-SNE feature visualizations



(a) USPS→MNIST



(b) VisDA

(left) source, target and mapped source. (right) classes in source and target.

# Performance on visual UDA datasets

Balanced accuracy ( $\uparrow$ ) over 10 runs with best performance in **bold**.  
Results are aggregated over imbalance scenarios and datasets.

Setting	Source	DANN	$WD_{\beta=0}$	$WD_{\beta=1}$	$WD_{\beta=2}$	MARSG	MARSc	IW-WD	OSTAR+IM
Digits									
balanced	74.98 $\pm$ 3.8	90.81 $\pm$ 1.3	92.63 $\pm$ 1.0	82.80 $\pm$ 4.7	76.07 $\pm$ 7.1	92.18 $\pm$ 2.2	94.91 $\pm$ 1.4	95.89 $\pm$ 0.5	<b>97.51 <math>\pm</math> 0.3</b>
subsampled	75.05 $\pm$ 3.1	89.91 $\pm$ 1.5	89.45 $\pm$ 1.0	81.56 $\pm$ 4.8	77.77 $\pm$ 6.5	91.87 $\pm$ 2.0	93.75 $\pm$ 1.4	93.22 $\pm$ 1.1	<b>96.69 <math>\pm</math> 0.7</b>
VisDA12									
original	48.63 $\pm$ 1.0	53.72 $\pm$ 0.9	57.40 $\pm$ 1.1	47.56 $\pm$ 0.8	36.21 $\pm$ 1.8	55.62 $\pm$ 1.6	55.33 $\pm$ 0.8	51.88 $\pm$ 1.6	<b>59.24 <math>\pm</math> 0.5</b>
subsampled	42.46 $\pm$ 1.4	47.57 $\pm$ 0.9	47.32 $\pm$ 1.4	41.48 $\pm$ 1.6	31.83 $\pm$ 3.0	55.00 $\pm$ 1.9	51.86 $\pm$ 2.0	50.65 $\pm$ 1.5	<b>58.84 <math>\pm</math> 1.0</b>
Office31									
subsampled	74.50 $\pm$ 0.5	76.13 $\pm$ 0.3	76.24 $\pm$ 0.3	74.23 $\pm$ 0.5	72.40 $\pm$ 1.8	80.20 $\pm$ 0.4	80.00 $\pm$ 0.5	77.28 $\pm$ 0.4	<b>82.61 <math>\pm</math> 0.4</b>
OfficeHome									
subsampled	50.56 $\pm$ 2.8	50.87 $\pm$ 1.05	53.47 $\pm$ 0.7	52.24 $\pm$ 1.1	49.48 $\pm$ 1.3	56.60 $\pm$ 0.4	56.22 $\pm$ 0.6	54.87 $\pm$ 0.4	<b>59.51 <math>\pm</math> 0.4</b>

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Domain-invariant baselines designed for:

- Covariate shift w/o reweighting (DANN Ganin et al. 2016,  $WD_{\beta=0}$  Shen et al. 2018).
- GeTarS with reweighting ( $WD_{\beta \in \{1,2\}}$  Wu et al. 2019, MARS Rakotomamonjy et al. 2021; IW-WD Combes et al. 2020).

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## Paper, code and contact

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- <https://github.com/mkirchmeyer/ostar>
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Thank you for your attention !

# References I

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