# UNSUPERVISED DOMAIN <br> ADAPTATION WITH NONSTOCHASTIC MISSING DATA 

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Introduction

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MRI Modality


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為楽在他，為豆在己。為槚在他，為之與㒒，詂之與講。故之與右，諾之者言，依於博，與博者言，依於解。劳。與富者言，依於豪。與省者言言，依於說。此言之術也。不用在早非所宜為，勿為以避其危。非所宜取避其緊。一登而非，䢂馬刎追。一言語不留耳。此謂君子也。夫任臣之法親也，勇則不近也，信則不信也。不

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| IS Ultra－Compact Binoculars |  | Tongass National Forest Map |  |
| :--- | :--- | :--- | :---: |
| Lightweight and powerful， <br> the ultra－compact $10 \times 30$ <br> Image Stabilization <br> Binoculars delivers（．．．） | Detailed Map Of Prince of <br> Wales Island in Tongass <br> National Forest．This Map <br> is detailed（．．．） |  |  |
| ＂Excellent Optics．＂ |  |  |  |


（c）

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■ Existing methods usually consider stochastic missing data．
－Missing Completely At Random（MCAR）Rubin 1976

$$
\forall x, p_{\phi}(\mathrm{m} \mid \mathrm{x})=p_{\phi}(\mathrm{m})
$$

m stochastic．

## Non-stochastic missing data

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## Contributions

- Handle non-stochastic missing data with unsupervised domain adaptation (UDA).

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## Contributions

- Handle non-stochastic missing data with unsupervised domain adaptation (UDA).
Formalize the problem.

1 labelled $x_{S}$ and unlabelled $x_{T}$ under distribution shift.

Source domain
Full and labelled


Target domain
Missing and unlabelled


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$2 \mathrm{x}_{\mathrm{e}}=\left(\mathrm{x}_{\mathrm{e}_{1}}, \mathrm{x}_{\mathrm{e}_{2}}\right), e \in\{S, T\}$ with $\mathrm{x}_{\mathrm{S}}$ fully observed; $\mathrm{x}_{\mathrm{T}_{2}}$ missing.

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Goal: train a classifier $\hat{h}$ with low classification error on $T$.

Model

## Model components

Model: $\hat{h}: \mathcal{X}_{1} \rightarrow \mathcal{Y}=\{0, \ldots, K\}, \hat{h}=f \circ \hat{g}$ on $\mathbf{S}$ and $\mathbf{T}$


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- $r: \mathcal{Z}_{1} \rightarrow \mathcal{Z}_{2}$ conditional generator of $\mathrm{z}_{\mathrm{e}_{2}}$ given $\mathrm{z}_{\mathrm{e}_{1}}$.



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Reference: $h: \mathcal{X} \rightarrow \mathcal{Y}, h=f \circ g$ only on $\mathbf{S}$

- $g: \mathcal{X}_{1} \times \mathcal{X}_{2} \rightarrow \mathcal{Z}$ encoder with both ( $\left(\mathrm{e}_{\mathrm{e}_{1}}, \mathrm{x}_{\mathrm{e}_{2}}\right)$ components.
- $g_{2}: \mathcal{X}_{2} \rightarrow \mathcal{Z}_{2}$ encoder of $\mathrm{x}_{\mathrm{e}_{2}}$.



## Model training

## Adversarial training

Three modules for imputation, adaptation, classification.

## Model training

## Adversarial training

■ Imputation: alignment loss $L_{2}=L_{A D V}+\lambda_{M S E} L_{M S E}$.


## Model training

## Adversarial training

- Adaptation: alignment loss $L_{1}$.



## Model training

## Adversarial training

- Classification: cross-entropy loss $L_{3}$.



## Model training

$$
\begin{equation*}
\min _{g_{1}, g_{\mathbf{2}}, r, f} \max _{D_{\mathbf{1}}, D_{\mathbf{2}}} L_{1}+\left(L_{A D V}+\lambda_{M S E} L_{M S E}\right)+L_{3} \tag{1}
\end{equation*}
$$



Formalization

## Assumptions

Conditional invariance
After projection with $g=\left(g_{1}, g_{2}\right)$,

$$
p_{S}\left(Z_{2} \mid Z_{1}\right)=p_{T}\left(Z_{2} \mid Z_{1}\right), \quad p_{S}\left(Z_{1}\right) \neq p_{T}\left(Z_{1}\right)
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We can use available supervision in $S$ to infer $p_{T}\left(Z_{2} \mid Z_{1}\right)$.

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## Upper-bounds

- Adaptation upper-bound of the target error of $\hat{h}$
- Imputation upper-bound of the target error of $h$


## Upper-bounds

## Adaptation upper-bound Ben-David et al. 2010

Given $f \in \mathcal{F}$ and $\hat{g}$

$$
\begin{equation*}
\epsilon_{T}(f \circ \hat{g}) \leq \underbrace{\left[\epsilon_{S}(f \circ \hat{g})+d_{\mathcal{F} \Delta \mathcal{F}}\left(p_{S}(\hat{Z}), p_{T}(\hat{Z})\right)+\lambda_{\mathcal{H}_{\hat{g}}}\right]}_{\text {Domain Adaptation (DA) }} \tag{2}
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$$

$L_{3} \rightarrow$ 1st term, $L_{1} \rightarrow$ 2nd term, Covariate Shift $\rightarrow$ 3rd term small.

## Upper-bounds

## Imputation upper-bound

Under Conditional Invariance, given $f, \hat{g}$ and $g$,

$$
\begin{align*}
\epsilon_{T}(f \circ g) & \leq \underbrace{\sup _{\mathrm{z} \sim p(Z)}\left[\frac{p_{S}\left(Z_{2}=\mathrm{z}_{2} \mid \mathrm{z}_{1}\right)}{p_{S}\left(\hat{Z}_{2}=\mathrm{z}_{2} \mid \mathrm{z}_{1}\right)}\right]}_{\text {Imputation error on T }\left(I_{T}\right)} \times \underbrace{\sup _{\mathrm{z} \sim p(Z)}\left[\frac{p_{S}\left(\hat{Z}_{2}=\mathrm{z}_{2} \mid \mathrm{z}_{1}\right)}{p_{T}\left(\hat{Z}_{2}=\mathrm{z}_{2} \mid \mathrm{z}_{1}\right)}\right]}_{\text {Tratation error on S }\left(I_{S}\right)} \\
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## Experiments

## Experimental setting

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- ZeroImputation: full $\mathrm{x}_{\mathrm{S}}$; missing $\mathrm{X}_{\mathrm{T}_{2}}$ set to $0, \mathrm{x}_{\mathrm{T}}=\left(\mathrm{x}_{\mathrm{T}_{1}}, \mathbf{0}\right)$.


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- Imputation: full $x_{S}$; missing $X_{T_{2}}$ imputed.
- Two divergences for aligning distributions:
- $\mathcal{H}$-divergence
- Wasserstein distance


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- IgnoreComponent: only $\mathrm{x}_{S_{1}}, \mathrm{x}_{\mathrm{T}_{1}} ; \mathrm{x}_{\mathrm{S}_{2}}, \mathrm{x}_{\mathrm{T}_{2}}$ ignored.
- Imputation: full $x_{S}$; missing $x_{T_{2}}$ imputed.
- Two divergences for aligning distributions:
- $\mathcal{H}$-divergence
- Wasserstein distance


## Datasets and Metrics

## Experimental setting

## Baselines

- Full: full $\mathrm{x}_{\mathrm{S}}$ and $\mathrm{x}_{\mathrm{T}}$.
- ZeroImputation: full $x_{S}$; missing $x_{T_{2}}$ set to $0, x_{T}=\left(x_{T_{1}}, 0\right)$.

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## Datasets and Metrics

- digits (missing half pixels): accuracy


## Experimental setting

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## Datasets and Metrics

- digits (missing half pixels): accuracy
- Amazon product reviews (missing half embeddings): accuracy


## Experimental setting

## Baselines

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## Datasets and Metrics

- digits (missing half pixels): accuracy
- Amazon product reviews (missing half embeddings): accuracy
- challenging real-world advertising datasets ${ }^{1}$ : cross-entropy

[^0]
## Results - Target accuracy $(\uparrow)$ and Cross-Entropy $(\downarrow)$

| Dataset | MNIST $\rightarrow$ USPS |  | USPS $\rightarrow$ MNIST |  | SVHN $\rightarrow$ MNIST |  | MNIST $\rightarrow$ MNIST-M |  | ads-kaggle | ads-real |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Model w/o $\mathscr{R}$ | ADV | OT | ADV | OT | ADV | OT | ADV | OT | ADV | ADV |
| Source-Full | $71.5 \pm 2.7$ |  | $74.2 \pm 2.7$ |  | $58.1 \pm 1.1$ |  | $28.3 \pm 1.4$ |  | NA |  |
| Adaptation-Full | $85.8 \pm 3.2$ | $92.6 \pm 1.7$ | $94.6 \pm 2.1$ | $93.9 \pm 0.6$ | $78.0 \pm 3.4$ | $76.1 \pm 1.4$ | $60.8 \pm 3.8$ | $46.9 \pm 3.9$ |  |  |
| Source-ZeroImputation | $25.7 \pm 3.7$ |  | $39.2 \pm 2.6$ |  | $31.5 \pm 2$. |  | $14.4 \pm 1.1$ |  | $0.545 \pm 0.019$ | $0.663 \pm 0.011$ |
| Adaptation-ZeroImputation | $48.4 \pm 4.8$ | $60.9 \pm 6.3$ | $67.5 \pm 2.2$ | $65.3 \pm 5.2$ | $47.1 \pm 5.7$ | $37.5 \pm 6.2$ | $34.7 \pm 2.5$ | $20.2 \pm 2.5$ | $0.397 \pm 0.0057$ | $0.660 \pm 0.025$ |
| Source-IgnoreComponent | $52.9 \pm 9.7$ |  | $54.3 \pm 1.6$ |  | $44.6 \pm 1.9$ |  | $19.1 \pm 2.6$ |  | $0.406 \pm 0.00046$ | $0.622 \pm 0.0048$ |
| Adaptation-IgnoreComponent | $71.5+3.2$ | $64.0 \pm 5.0$ | $80.0 \pm 1.4$ | $72.0 \pm 1.8$ | $45.5 \pm 1.9$ | $47.9 \pm 1.8$ | $29.4 \pm 1.6$ | $26.8 \pm 4.4$ | $0.403 \pm 0.0030$ | $0.634 \pm 0.0082$ |
| Adaptation-Imputation | $74.2 \pm 2.3$ | $66.8 \pm 1.3$ | $\mathbf{8 1 . 4} \pm \mathbf{0 . 8}$ | $72.5 \pm 2.7$ | $53.8 \pm 1.4$ | $49.2 \pm 1.5$ | $57.9 \pm 2.3$ | $29.2 \pm 1.4$ | $\mathbf{0 . 3 8 9} \pm 0.014$ | $\mathbf{0 . 5 8 3} \pm 0.013$ |


| Dataset | DVD $\rightarrow$ Electronics | Books $\rightarrow$ Kitchen | Kitchen $\rightarrow$ Electronics | DVD $\rightarrow$ Books |
| :---: | :---: | :---: | :---: | :---: |
| Source-Full | 69.57 | 73.04 | 77.88 | 71.95 |
| Adaptation-Full | 73.62 | 74.09 | 79.63 | 72.65 |
| Source-ZeroImputation | 58.51 | 60.52 | 66.27 | 61.15 |
| Adaptation-ZeroImputation | 64.51 | 61.08 | 68.02 | 62.80 |
| Source-IgnoreComponent | 60.21 | 62.03 | 67.62 | 64.35 |
| Adaptation-IgnoreComponent | 61.02 | 64.08 | 68.47 | 66.00 |
| Adaptation-Imputation | $\mathbf{7 2 . 5 7}$ | $\mathbf{7 2 . 6 9}$ | $\mathbf{7 8 . 1 8}$ | $\mathbf{7 2 . 6 1}$ |

## Conclusion

## Our model improves representative baselines:

- on all our datasets
- for two alignment divergences


## Ablation studies - Model modules

| Ablation study | ADV Model | MNIST $\rightarrow$ USPS | USPS $\rightarrow$ MNIST | SVHN $\rightarrow$ MNIST | MNIST $\rightarrow$ MNIST-M | ads-kaggle |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L_{2}+L_{3}$ vs. $L_{1}+L_{2}+L_{3}$ | $L=\lambda_{2} L_{2}+\lambda_{3} L_{3}$ | $64.2 \pm 1.8(-13 \%)$ | $51.3 \pm 2.5(-37 \%)$ | $44.5 \pm 1.4(-17 \%)$ | $24.1 \pm 2.6$ (-58\%) | $0.410 \pm 0.0020$ (-5.4\%) |
| ADV-MSE weighting in $L_{2}$ | $L_{2}=L_{M S E}$ | $71.9 \pm 3.7$ (-3.1\%) | $81.4 \pm 1.2$ (0\%) | $52.5 \pm 3.7(-2.4 \%)$ | $56.5 \pm 2.8$ (-2.4\%) | $0.400 \pm 0.0014(-2.8 \%)$ |
|  | $L_{2}=L_{\text {ADV }}$ | $28.6 \pm 3.2(-61 \%)$ | $39.4 \pm 5.2(-52 \%)$ | $28.8 \pm 3.8$ (-46\%) | $30.0 \pm 3.7(-48 \%)$ | $0.469 \pm 0.13$ (-21\%) |
|  | $L_{2}=L_{A D V}+0.005 \times L_{M S E}$ | $47.8 \pm 3.7(-36 \%)$ | $49.6 \pm 5.8(-39 \%)$ | $46.0 \pm 2.6$ (-15\%) | $50.6 \pm 2.2(-13 \%)$ | $0.389 \pm 0.014$ (0\%) |
|  | $L_{2}=L_{A D V}+L_{M S E}$ | $74.2 \pm 2.3$ (0\%) | $81.4 \pm 0.8$ (0\%) | $53.8 \pm 1.4(0 \%)$ | $57.9 \pm 2.3(0 \%)$ | $0.401 \pm 0.0014(-3.1 \%)$ |
| Ablation study | ADV Model | DVD $\rightarrow$ Electronics | Books $\rightarrow$ Kitchen | Kitchen $\rightarrow$ Electronics | DVD $\rightarrow$ Books |  |
| ADV-MSE weighting in $L_{2}$ | $L_{2}=L_{M S E}$ | 71.47 (-1.5\%) | 71.39 (-1.8\%) | 77.58 (-0.77\%) | 72.02 (-0.81\%) |  |
|  | $L_{2}=L_{A D V}+L_{M S E}$ | 72.57 (0\%) | 72.69 (0\%) | 78.18 (0\%) | 72.61 (0\%) |  |



Figure 1: Adaptation-Imputation T CE $(\downarrow)$ on ads-kaggle wrt $\lambda_{\text {MSE }}$

## Conclusion

- $L_{1}$ is useful.
- $L_{A D V}$ in $L_{2}$ is useful.

Conclusion

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## Problem

New end-to-end approach for non-stochastic missing data based on an adaptation-imputation problem.

## Conclusion

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New end-to-end approach for non-stochastic missing data based on an adaptation-imputation problem.

## Theory

Clear assumptions and upper-bounds minimized by our model.

## Conclusion

## Problem

New end-to-end approach for non-stochastic missing data based on an adaptation-imputation problem.

Theory
Clear assumptions and upper-bounds minimized by our model.

## Experiments

Superior performance over representative baselines on real-world datasets with extremely different characteristics.

## Conclusion

## Thank you for your attention!

Code: https://github.com/mkirchmeyer/adaptation-imputation Contact information:

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References

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[^0]:    $1_{\text {http://labs.criteo.com/2014/02/kaggle-display-advertising-challenge-dataset/ }}$

